Integration by Parts

Math 130 - Essentials of Calculus

30 April 2021

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This is the formula for integration by parts.

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We typically do integration by parts by choosing a u and a dv then applying this formula. So how do we choose u and dv? There's actually a good way to choose, and the general idea is we want to choose u so that du becomes simpler, then pick dv to be everything else in the integral.

Choosing u and dv

There's a nice acronym we can use as a guide for how to choose u and dv. We pick u to be the first thing that appears in this list:

- **L**ogarithmic
- **I**nverse Trigonometric*
- **A**lgebraic
- **T**rigonometric
- **E**xponential

(*we won't use these in this class)

Choosing u and dv

So really, for this class we can just think of the acronym as

- Logarithmic
- Algebraic
- Trigonometric
- Exponential

Examples

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4. $\int \ln x \, dx$
5. $\int x^2 e^x \, dx$

Integration by Parts for Definite Integrals

Integration by parts with definite integrals works very cleanly:

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Compute the integrals

1. $\displaystyle\int_0^1 xe^{4x}\, dx$

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Example

Compute the integrals

1. $\int_0^1 xe^{4x}\ dx$
2. $\int_1^2 t^4 \ln t\ dt$
3. $\int_1^2 \frac{\ln z}{z^2}\ dz$
4. $\int_0^5 ye^{-0.6y}\ dy$